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REVIEWS OF BOOKS.

La logique de Leibniz d'après des documents inédits. Par LOUIS COUTURAT. Paris, Alcan, 1901. — pp. xiv, 608.

The body of this work (pp. 1–441) consists of a short critical Conclusion (pp. 431–441), preceded by nine expository chapters dealing with the following subjects: (1) The Syllogistic, (2) the 'Combinatoria,' (3) the Universal Language, (4) the Universal Characteristic, (5) the Encyclopædia, (6) the 'General Science,' (7) the Universal Mathematics, (8) the Logical Calculus, (9) the Geometrical Calculus. Then follow (pp. 443–538) five Appendices: (1) Abstract of the Classical Logic, (2) Leibniz and Hobbes, their Logic, their Nominalism, (3) Some Mathematical Discoveries of Leibniz, connected with the Combinatoria and the Characteristic, (4) On Leibniz as Founder of Academies, (5) On the Geometrical Calculus of Grassmann. The work closes (pp. 539–608) with twenty notes, a table of bibliographical abbreviations, a table of correspondence between the editions of Gerhardt and Erdmann, indexes, etc.

The book is, from every point of view, a noteworthy one. It is the product of an author who is at once — a rare combination — a trained mathematician, logician, and philosopher. It is the result of laborious and scholarly research, including a careful study of many important unpublished writings of Leibniz, neglected even by Gerhardt. Out of a great wealth of material all pertinent facts have been seized upon, admirably marshalled without apparent distortion or undue emphasis, and the whole presented in a charmingly lucid style. And finally, the conclusions reached are revolutionary; if accepted, the traditional interpretation of much of the contents of Leibniz's philosophy and the usual account of its evolution must be discarded.

The book originated in an attempt to make a study of Leibniz as a precursor of modern Symbolical Logic, to analyze his Logical Calculus and his Geometrical Calculus, and to reconstruct the conception of his Universal Characteristic. In carrying out this design, M. Couturat was led to examine the unpublished writings of Leibniz contained in the library at Hanover, being greatly aided in doing so by Herr Bodemann's catalogue. He found that the various editors of Leibniz's works, such as Raspe, Erdmann, and even Gerhardt, had particularly neglected in their editions the logical writings; that, for one manuscript on logic which they had published, they had left a score of others, just as important and as finished, if not more so, unpublished; and, what is most remarkable, nearly all the dated manuscripts. M. Couturat's explanation of this astonishing fact is doubtless the true one, viz., that Leibniz's editors did not understand these fragments on logic and were unable to appreciate their value.

The revolutionary conclusion to which the author comes (pp. vii f.; 431 f.), the central and essential outcome of the work before us, is that above all things else and throughout his life Leibniz was a logician, and his logic was the real foundation of his whole system. His philosophy reposes solely upon the principles of his logic and proceeds entirely from them. His logic is at once the source, the center, and the connecting bond of his metaphysical speculations and of his mathematical discoveries. Both the Monadology and the Infinitesimal Calculus rest upon and flow from it. Thus, according to M. Couturat, Leibniz's logic was not only the heart and soul of his system, but the center of all his intellectual activity and the source of all his discoveries, "le foyer obscur, ou du moins caché, d'où jaillirent tant de lumineuses 'fulgurations'" (p. xii).

The work, however, is not a piece of special pleading; far otherwise. The revolutionary conclusion to which it comes as to the capital place which Leibniz's logic occupies in his system, was, we are told, neither sought by the author nor even foreseen by him; it was forced upon him almost in spite of himself. The calm and unprejudiced manner in which the body of the book is written fully bears out this statement. The work is characterized throughout by an absence of all apparent special pleading, and of any detailed criticism. It is preëminently, what its author calls it, a *purely historical work* (p. 431), a calm exposition of what he believes he finds in Leibniz's own writings, supported by copious references to, and quotations from, the unpublished manuscripts as well as from the published writings.

M. Couturat's book, however, apart from the importance of the conclusions to which it comes regarding the relation of Leibniz's logic to the rest of his system, is of great value on account of the full account it gives of Leibniz's various logical labors, of their historical development, and of their connection one with another. All students of Leibniz have known that he was interested, especially in his youth, in schemes for improving logic, and in schemes for a sort of universal language. Expositors and critics of his philosophy have, however, usually regarded schemes of the latter sort as quite visionary, and have given little attention to them. Credit has always, however, been given to Leibniz for a certain amount of valuable work in logic. The law of sufficient reason; the criteria for determining the quality of terms — their clearness, distinctness, and adequacy; certain forms of immediate inference, such as the inversion of relations and inferences from added determinants; certain imperfectly understood suggestions toward a Symbolical Logic, and the like, have usually been given as the sum of his logical achievements. But it is safe to say that heretofore his real aims, the extent of his interests, and his actual achievements in logic have never been fully understood or properly appreciated. In M. Couturat's book we have the first adequate account of them. In spite of the fact that many of these labors were abortive, there is a large amount of successful work and a wealth of fruitful suggestion; and M. Couturat's book cannot fail to give one a fresh and vivid impression of Leibniz's genius and of his astonishing versatility.

In his first chapter, M. Couturat gives a brief account of Leibniz's views of the traditional logic of Aristotle and the Schoolmen, especially of the syllogism and of his suggestions towards its improvement. Attention is called at the outset to the well-known passage in the *Nouveaux essais* (IV, xvii, 4) in which Leibniz declares that the form of the syllogism is one of the most beautiful and most considerable discoveries of the human mind, and that it is a *sort of universal mathematics.* Leibniz, it is shown (p. 4), reduced propositions as to quantity from the traditional four to the now accepted two; he adopted (p. 8) the ingenious method, suggested by Ramus and Thomasius and now known as *reductio ad absurdum,* of re-reducing the modes of the last three figures to the first; he demonstrated (p. 49 f.) syllogistically subalternation and conversion; he vigorously defended the fourth figure against its critics; he anticipated (pp. 21, 25) Euler in the use of circles by which to represent the meaning of propositions, and also invented a more ingenious and more perfect linear symbolism. He appears, however, to have constantly oscillated between the intensive and the extensive interpretation of propositions, while showing a marked predilection, although apparently more instinctive than reasoned, for the former.

In Chapter II, we have an account of Leibniz's youthful production, published in his twentieth year (1666), *Dissertatio de arte combinatoria,* and of its relations to his later projects. In this essay Leibniz held (p. 49) that all complex concepts or ideas can be resolved into simple ones, by an analysis analogous to the resolution of numbers into their prime factors; and, conversely, that all complex ideas can be obtained and composed by the progressive combination of these simple ones. The simple concepts or ideas, the constitutive elements of all the others, are few in number, and to each of them a name or sign should be assigned, and thus the *alphabet of human thoughts* would be formed. Here, according to M. Couturat (p. 431), we have the two fundamental postulates of Leibniz's logic, *viz.:* "(1) All our ideas are composed of a very small number of simple ideas, which together form the *alphabet of human thoughts;* (2) Complex ideas proceed from these simple ideas by a uniform and symmetrical combination analogous to arithmetical multiplication." Both of these postulates are declared (pp. 431, 432) by M. Couturat to be false; but nevertheless the importance of the *De arte combinatoria* in the evolution of Leibniz's thinking and in the formation of his logic can hardly be overestimated, for the Combinatoria translated the combinations of simple ideas by the help of the signs or characters which symbolize them; and thus we are led to construct a Universal Characteristic; and from that we are led to form a Symbolical Logic which will substitute for ideas the combination of signs or characters, for propositions the relations between these symbols, and for inference a sort of calculus, and thus furnish a universal and infallible method for demonstrating known propositions and for discovering from them new ones. Leibniz himself later refers to the *Combinatoria* as the prelude and the anticipation of his further discoveries; and he seems to have held to the end of his life that its basis was sound. M. Couturat finds (p. 49, note) among the unpublished manuscripts of Leibniz two later plans for a new *De arte combinatoria*, from which it appears that the conception in its details underwent considerable modification in the mind of Leibniz.

Chapter III deals with Leibniz's various projects for a universal spoken or written language, the form under which he at first conceived his Characteristic. He seems to have been quite familiar with the different contemporary projects of Becker, Kircher, and others. These projects hardly deserve to be called schemes for a universal language, resembling as they do more nearly modern diplomatic ciphers or telegraphic codes, and resting on no logical and philosophical basis. The ambiguity of terms, giving a term several meanings in each language; the lack of exact synonyms, rendering exact correspondence between the words of two languages impossible; the diversities of syntax, rendering literal translations often unintelligible, make these earlier projects impracticable. At the age of twenty, Leibniz conceived a plan of a universal language which should be truly philosophical and rest upon a logical basis; *viz.*, a complete analysis of concepts or ideas and their reduction to simple ones, each simple term to be represented by as natural and simple a sign or character as possible. In this way

Leibniz would form a sort of ideographic alphabet composed of as many symbols as there are elementary ideas. In a word, the scheme was for an *ideography*, rather than for a *stenography*. Stimulated more or less by the projects of Wilkins and Dalgarno, Leibniz labored to develop and to perfect his project. A *real* Characteristic is for him an *ideography*, that is, a system of signs immediately representing things (or rather ideas), and not words; so that the people of each nation could read them and translate them into their own language (p. 61). Leibniz came, however, to see that the problem was less simple than he had at first supposed; and, in prosecuting it further, he adopted an *a posteriori* method. He took the living languages as his point of departure, and by a logical analysis undertook to derive from them, on the one hand, the simple ideas, and, on the other hand, a rational or philosophical grammar. He began working on a rational grammar as early as April, 1678; and primarily applied it to the Latin as a basis or intermediary. All exceptions and inflections, and all distinctions of number and gender, were eliminated. He rejected the Aristotelian view that the office of the verb is to express time. He held that the noun expresses an idea; the verb a proposition (affirmation or negation). All speech is finally (p. 70) reduced to "the one noun *ens*, to the one verb *est*, to adjectives (expressing qualities), and to particles which serve to connect all the preceding words and to indicate their relations." He attempts even to carry further the analysis of particles (pp. 71 f.). These studies in comparative philology and grammatical analysis all had a logical aim. In the *Nouveaux essais* (III, vii, 6) he writes: "I truly believe that language is the best mirror of the human mind, and that an exact analysis of the meaning of words would reveal better than anything else the workings of the understanding." Such a grammatical analysis reveals primitive logical relations (p. 76) and enables us to demonstrate certain 'asyllogistic' inferences (p. 75), such as the inversion of relations, *e. g.*, Peter resembles Paul, therefore Paul resembles Peter. The ideal, indeed, of the universal language is to express concepts by characters which will make manifest both their composition *and their relations* (p. 75).

In order to constitute the *alphabet of human thoughts*, which is to serve as the basis of the universal language, there is need of an inventory of human knowledge, a demonstrative encyclopædia; and at the same time that our primitive concepts are enumerated and classed, they must be represented by appropriate characters, and signs must be invented to express their combinations and their relations — the work of the Universal Characteristic (p. 79). Thus the true universal, or rather philosophical, language presupposes the Encyclopædia and the Characteristic; and these two must be elaborated together, for the one involves the other (p. 80). In considering them, however, we may begin with the Characteristic (Ch. IV).

Leibniz understands, as we have seen, by *real* characters, signs which directly represent, not words, letters, or syllables, but things, or rather ideas. Among these signs he recognizes an important difference; some represent ideas only, others assist in reasoning. Egyptian and Chinese hieroglyphics, and the astronomical and chemical symbols, are examples of the first class; arithmetical marks and algebraic signs, of the second. Characters of the latter sort — to aid reasoning — are what Leibniz wanted for his Characteristic. Hence he regarded arithmetic and algebra as samples of his Characteristic, and as proofs of its possibility and value. The capital advantage which he attributed to his Characteristic was that it would reduce all human reasoning to a sort of calculus (p. 84 n.), enabling us to reason by a calculus analogous to those of arithmetic and algebra. Mathematics owes its progress, according to Leibniz, to the fact that it possesses suitable symbols in its arithmetical and geometrical signs; and his own progress in mathematics he himself attributed to the fact that he had succeeded in finding appropriate symbols with which to represent quantities and their relations (p. 84). His discovery of the Infinitesimal Calculus grew out of his constant search for new symbolism (pp. 84, 87), and in turn contributed much to confirm him in his estimate of the great importance of a good characteristic for the deductive sciences (p. 84). The conditions of a good characteristic (pp. 87–89), if realized, would, he thinks, give us a logical calculus which would be superior to the Cartesian method, and would be the real geometrical method sought for in vain by Descartes and Spinoza. Among the many uses of the Characteristic, Leibniz boasts especially that it would put an end to disputes and the interminable discussions of the schools (p. 97); he speaks of it, in fact, as "un juge des controverses veritablement infallible" (p. 98 n.). To settle a question, philosophical or otherwise, or end a controversy, future disputants would only need to take their pens, call in a friend to act as umpire, and say 'Calculemus!' (p. 98). His Characteristic was to be applicable to all domains of knowledge, prevent error, discover truth, be the *organon* or instrument of reason, and, in fact, be an actual substitute for reasoning (p. 101). Leibniz by turns employed various schemes of symbols, arithmatical, algebraical, geometrical, and even mechanical. These he seems to have admitted as parallel and equivalent systems (p. 116).

In addition to the difficulty of settling upon a definitive symbolism, the realization of Leibniz's great project for the Universal Characteristic was retarded, he himself tells us, by the fact that it presupposed the elaboration of the Encyclopædia, or at least a collection of logical definitions of all the fundamental concepts of the different sciences (p. 117).

In Chapter V, we have an interesting account of the various projects for an Encyclopædia which Leibniz entertained during the course of his career, and of the reasons why his far-reaching enterprise failed of realization. This Encyclopædia was to have been Leibniz's great philosophical and scientific work. It was to have been (p. 119) "an abstract or summary of all human knowledge, historical and scientific, arranged in a logical order and following a demonstrative method, commencing with definitions of all the simple and primitive terms (those forming the *alphabet of human thoughts*)." As a result, all the sciences would assume the deductive type (pp. 152, 154), and all of them would rest upon a small number of principles or hypotheses (p. 152). They would also "be abridged in being augmented" (p. 152). The very earliest essays of Leibniz relating to this project for an encyclopædia reveal the fact that he was even then deeply interested in all orders of knowledge with a curiosity truly universal, and that he aimed to introduce into all the sciences, even the moral and practical sciences, the logical clearness and the demonstrative force which he thought he found in mathematics. At one time he plans to found a Bibliographical Review which should give an account of all new books; at another, he conceives of a sort of portable library, an *Opus Photianum* (p. 123), giving the substance of all literature; at another, he plans the correction and completion of the *Encyclopædia* of Alsted, which had appeared about 1671. In 1676 he aims to found a scientific society which should enlist men of learning generally in advancing the sciences; the outcome of their joint labors would be the Encyclopædia. An *Imperial German Society*, under the patronage of the Emperor, is also outlined (p. 127); and until the very end of his life Leibniz labored without ceasing on an astonishing number of similar projects (see the interesting Appendix IV, "Leibniz as Founder of Academies"), turning now to this, now to that sovereign, and to individual scholars and to the learned generally, for help in realizing them. And in all these projects he never lost sight of his scheme for the Encyclopædia. In this connection, also, he occupied himself throughout life in finding or collecting *definitions* of all sorts, attaching great value and logical importance to good definitions. He speaks of having "quantité de définitions" (p. 169), especially in logic, in metaphysics, and in ethics. Numerous long tables of definitions, five of especial importance (p. 170), are found among his manuscripts.

This vast project of an Encyclopædia, cherished through a long life, was never realized. Leibnitz himself gives (p. 175) two reasons for this — lack of collaborators, and lack of time.

The elaboration of the Encyclopædia presupposes a *Scientia generalis*, that is to say, a universal method applicable to all the sciences. This general science, to which M. Couturat devotes over one hundred pages (Ch. VI), constituted the whole of Leibniz's logic. Logic, in the most comprehensive sense, consists, according to Leibniz, of two parts. The first is the art of judgment and proof, is the method of certainty, and serves to demonstrate truths already discovered and to verify doubtful or contested propositions. It proceeds from principles to consequences, from causes to effects; is progressive and synthetic. The second is the art of discovery; aims to discover new truths by a sure and nearly infallible method and in a systematic order. It proceeds from consequences to the principles required, from known effects to unknown causes; is regressive and analytic. The two parts thus resemble synthesis and analysis in geometry. Later, however, Leibniz came to see that analysis and synthesis are alike employed both in demonstration and in discovery and together form but one method. The real difference, therefore, between the two parts of logic is found rather in the purpose with which the one method is employed, and in the purely subjective fact that the truth to be established is in the one case known and in the other case unknown.

The *Scientia generalis* is thus a generalization of the method of mathematics (p. 179). Analysis consists in decomposing all concepts into their simple elements by means of definition; synthesis consists in reconstructing them by means of the art of combination. Thus the *De arte combinatoria* is the germ and principle of this logic and furnishes the key to this double process of analysis and of synthesis. But fortunately analysis need not be carried on indefinitely; for "the analysis of a truth is finished when its demonstration is found, and it is not always necessary to finish the analy is of the subject or predicate of a proposition in order to find its demonstration" (p. 183).

At first Leibniz affirmed that demonstration is nothing but a chain of definitions. Hence all axioms ought to be demonstrable. But the certainty of axioms does not come from experience, for induction could never justify a universal and necessary proposition (p. 185). Hence it rests upon the principle of identity or contradiction, the sole *a priori* principle that Leibniz recognizes. He boldly declares (p. 185) that all truths must be capable of demonstration, except *identical* propositions (those reducible to the principle of identity) and empirical propositions (those known by experience). All truths are first reduced to definitions, identical propositions, and empirical propositions; and finally to definitions and the principle of identity. All demonstration rests upon definitions and the axiom of identity (pp. 184 f., 203 f.).

The necessity belonging to axioms is found in the principle of contradiction or identity. The necessary is that whose contrary implies contradiction, "qui est verus atque unicus character impossibilitatis" (p. 187). Axioms are proved *by means* of definitions, but their truth does not rest upon the definitions but upon the principle of identity (p. 187). In contrast with the nominalism of Hobbes, Leibniz established a difference between nominal and real definitions which is of capital importance in his theory of knowledge. Only those definitions are *real* which make manifest the *possibility* of the thing defined, *i. e.*, the absence of all inner contradiction in the concept. Hence a real definition is not arbitrary (as with Hobbes), for it conforms to a true 'essence,' a possible 'nature' which does not depend upon us. Leibniz, therefore, rejects the scholastic rule for definition and substitutes for it one which may be formulated mathematically thus: "The definition ought to comprise the conditions necessary and sufficient to demonstrate all the properties of the object defined." This whole theory of definition starts from the *De arte combinatoria;* for to reach a perfect definition of a notion the analysis of the notion must be complete. When the notion has been resolved into its elements, the slightest contradiction becomes apparent and destroys the concept; hence an *adequate* notion is necessarily a true one, for all simple ideas are compatible among themselves. Leibniz's famous critique of the ontological argument, of so much consequence in his metaphysics, proceeds directly from these logical theories (pp. 195, 196, and note). His theory of definition is also of capital importance in his theory of knowledge, as expounded in his *Meditationes de cognitione, veritate, et ideis*, with its doctrine of clear, distinct, and adequate ideas. The complete analysis of truths and notions is indeed the ideal goal of science (p. 200). Its realization would establish the unity of science and furnish a rational basis and justification of the special sciences. The demonstration of axioms (other than the principle of identity) would contribute to this end; but, more important still, it would contribute toward the completion of the analysis of ideas and the discovery of the truly primary notions needed for the composition of the *alphabet of human thoughts* (p. 200). The insufficiency of the Cartesian rules of method, according to Leibniz, is due to the fact that they are psychological precepts, not logical ones, and consequently have a subjective rather than an objective reference (p. 202). The sole remedy for our errors is to be found in a good logic.

As all demonstration consists in substituting the definition for the thing defined, that is to say, in replacing a complex term by its equivalent group of more simple terms, the essential basis of deduction is *the principle of the substitution of equivalents*. This, and not the *Dictum de omni et nullo* of Aristotle, is the supreme and only principle of logic. Thus Leibniz anticipated Jevons's doctrine, "*The Substitution of Similars the true Principle of Reasoning*" (p. 206).

Not only all necessary truths, but all propositions whatsoever are analytic. In every true proposition the predicate is contained in the subject (pp. 208 f., and notes). "In saying," wrote Leibniz in 1668, "that the notion of Adam contains all that will ever happen to him, I am saying only what all philosophers understand in saying *prædicatum inesse subjecto veræ propositionis*." "That logical thesis," declares M. Couturat (p. 209 n.), "is the foundation of the whole of Leibniz's metaphysics." But if all propositions are thus analytic, what becomes of Leibniz's distinction between truths of reason (necessary truths) and truths of fact (contingent truths)? Are not the latter just as necessary as the former? The answer which Leibniz makes is that truths of fact are probable only *for us*, and because of our incomplete and merely approximate knowledge; in themselves, like the truths of reason and on the same ground and in the same degree, they are absolutely certain, for like them they are analytic or virtually identical. Like them, too, they are evident *a priori*, at least to an infinite intelligence which can grasp all their constitutive conditions. Herein consists properly the *principle of sufficient reason*, which Leibniz seems to have stated as early as 1670 (p. 214). The exact logical meaning of that famous principle, ordinarily stated as "Nothing is or happens without a reason," is "All truths are analytic," or "In every true proposition the notion of the predicate is contained in that of the subject" (pp. 214, 215). The principle is thus the converse of the principle of identity: "The principle of identity affirms that every identical proposition is true; the principle of sufficient reason that every true proposition is analytic, that is, virtually identical" (p.

215). These two complementary principles are inseparable, and are equally valid for all sorts of truths, for "it may be said," Leibniz declares (p. 217), "in a way, that these two principles are contained in the definition of the True and the False." The principle of identity, however, may be regarded as applying more particularly to truths of reason, and the principle of sufficient reason as applying more particularly to truths of fact, which latter cannot be justified without it (p. 217).

The necessary for Leibniz being that the opposite of which involves a contradiction, there is for him no necessity but logical necessity, and no impossibility but the logically contradictory (p. 219). But although whatever is not in itself contradictory is possible, all possibles cannot be realized together, for they are not all *compossible*, *i. e.*, mutually compatible. "It is as yet unknown," he tells us (p. 219 n. 2), "whence the incompossibility of different things springs, or what can make different essences to be opposed to each other, since all purely positive terms seem to be compatible." According to M. Couturat (pp. 219 n. 2, 432), Leibniz's difficulty here is to be attributed to his failure to take due account of logical *negation*. The principle of contradiction, Leibniz tells us, is the law of possibility or essence; the principle of reason, or of "the best," is the law of compossibility or existence. The principle of reason, purely logical in origin, thus assumes a metaphysical and theological character; and, applied to causality, a cosmological character. It enables us to find in God the 'first' or 'ultimate reason' of things. The laws of nature are contingent propositions depending upon the principle of reason, the free choice by God of 'the best.' By the principle of reason, as is well known, Leibniz rehabilitated the use of the principle of finality in physics. And his famous axioms, which play such a rôle in his physics and metaphysics, such as the axiom of symmetry, the principle of economy, the principle of the identity of indiscernibles, and the law of continuity, are (pp. 227 f.) but special applications of the principle of reason. Leibniz energetically and persistently affirms the perfect and absolute intelligibility of the world. For him things are hung on principles and have a logic running through them, are permeated by a sort of divine mathematics (p. 227); and God is "*avant tout* le grand calculateur et l'éternel logicien" (p. 237). "Cum Deus calculat et cogitationem exercet, fit mundus" (p. 227 n. 2).

Leibniz persistently deplored the absence of a satisfactory logic of probability, and himself made some valuable contributions towards its realization (pp. 248 f.). The theory of probability was regarded by Leibniz (p. 249) as the natural complement of the logic of certainty. It forms, indeed, the essential part of the logic of discovery; so much so that Leibniz in places substitutes the one for the other and gives as the two parts of logic the Logic of Certainty and the Logic of Probability, instead of the Logic of Certainty and the Logic of Discovery. The theory of probability plays an important rôle in the mathematical and rational sciences; but it is especially applicable to the natural and experimental sciences (p. 255), being their proper method. The sciences of nature deal with truths of fact, and the laws of nature are contingent truths. These truths and laws are known *a posteriori* and by experience. Two important truths of fact are the basal experiences, primitive data of consciousness, *I think*, and *I think different things*. They both testify to the actual existence of things: the first, as Descartes showed, reveals our own existence; the second, what Descartes failed to see, reveals the existence of an outer world (pp. 257, 258).

Leibniz's view of induction (pp. 261–271) is extremely interesting. Induction, as understood by empiricists, Leibniz condemns absolutely as insufficient and as even misleading (pp. 261, 262). It is unscientific and without logical value. In what, then, does the demonstration of a truth of fact, or of an empirical law, consist? It consists in deducing it from a more general hypothetical law, which may serve as the principle of other empirical laws; and in ascending thus progressively to laws more and more general, in such a way as to make all empirical laws depend upon the smallest possible number of principles or hypotheses. Thus the sciences of nature must be constructed upon the same deductive type as the rational sciences, and the method of the physical sciences be assimilated to that of mathematics; the only difference being that in the one case we proceed regressively and analytically, and in the other progressively and synthetically (pp. 264, 265). In fine, abstract mathematics is the true logic of the natural sciences, and the only experimental method is deduction, direct or inverse (p. 271).

An hypothesis is the more probable, according to Leibniz, (1) the more simple it is; (2) the greater the number of phenomena it explains with the smallest number of assumptions; and (3) the more insight it gives us into new phenomena and the better it explains new experiences. The world of phenomena is a great cryptogram, the keys to which are the laws of nature; such a law becomes more probable the greater the number of words and phrases it enables us to decipher. Thus the experimental sciences employ deduction in its two forms, the logic of certainty and the logic of probability, or the Characteristic and the Calculus of Probabilities, which latter is the true inductive method (p. 274).

In Chapter VII, M. Couturat seeks to determine Leibniz's conception of universal mathematics and the relation which logic and mathematics held in his thought. Mathematics served Leibniz as the model for his logic and was its inspiration. Mathematics, as traditionally conceived, has as its subject-matter magnitude and number. Mathematics, as conceived by Leibniz, has for its subject-matter not only number and magnitude, but whatever in the domain of sensible intuition is susceptible of exact and precise determination; it is, according to his expression, the *logic of the imagination*. Now the objects of the imagination are quality (or form) as well as magnitude; as to magnitude, things are equal or unequal; as to quality, they are like or unlike. The consideration of likeness, then, is as general and as fundamental as that of equality and belongs as essentially to universal mathematics. Consequently universal mathematics embraces two principal branches. The one is the science of magnitude (or equality) and its relations and proportions; that is, the traditional mathematics, summed up in logistic. The other is the science of forms (or of likeness), of order and of position; that is, the Combinatoria (pp. 290, 291). Mathematics in the traditional sense, *i. e.*, logistic or mathematical analysis, is subordinate to the Combinatoria, and the latter to logic itself (p. 299). Leibniz, therefore, long before modern discoveries and progress had revealed the fact, perceived that there is a universal mathematics, from which all the special mathematical sciences derive their most general principles and theorems, and that this universal mathematics is identical with logic itself, or at least is an integral part of it (p. 317). Universal mathematics is then the general science of relations (p. 317); but as a true formal logic is the science of all the general laws and forms of thought, the two coincide. Thus Leibniz conceived his logic as a mathematics of thought, or, as he expressed it, a "Universal Algebra"; something which has been realized by Whitehead in his *Treatise on Universal Algebra* (pp. 319, 320).

Side by side with the traditional algebra, which is nothing but the logic of number and magnitude, based on the single relation of equality, Leibniz conceived other algebras based on relations of congruence, similarity, etc. M. Couturat points out (p. 437) that Leibniz had "all the elements, or at least the materials of a Logic of

Relatives" ; and that (p. 303) he may justly be considered as the precursor of the Logic of Relatives as developed in more recent times by De Morgan, C. S. Peirce, and Schröder.

Of all the theoretically possible algebras included in universal mathematics, Leibniz attempted to elaborate but two: the logical calculus, which consists in the theory of identity and inclusion, and is applicable to both logic and geometry; and a geometrical calculus, immediately applicable to the study of spatial relations and embracing principally the theories of congruence and likeness (p. 321). Both proceed from the idea of the characteristic and are but two applications of it (p. 321).

In Chapter VIII, M. Couturat gives us an extremely interesting account of Leibniz's various labors in the province of symbolic logic (the logical calculus or algebra of logic). He shows that Leibniz occupied himself with this subject principally at three periods, 1679, 1686, 1690; from each of which we have a series of essays which taken together show that Leibniz was in possession of nearly all of the principles of the Algebra of Logic of Boole, Peirce, and Schröder, and on certain points was in advance of Boole himself (p. 386). He formulated the law, not only like Boole for logical multiplication, but also like Jevons for logical addition (p. 344). The important idea which constitutes perhaps Boole's finest discovery, Leibniz had clearly grasped as early as 1686; viz., the perfect analogy between categorical propositions and hypothetical ones, or, as Leibniz said, between incomplex terms and complex ones, that is, between concepts and propositions (p. 354). Leibniz had a more or less accurate idea not only of logical multiplication, addition, and negation, but also of logical subtraction and division. He knew the fundamental relations of the two copulas. He had found the true algebraical expression of the four classical propositions under their two principal forms. He had discovered the principal laws of the logical calculus, notably the rules of composition and of decomposition. Finally, he had a very clear conception of the double interpretation of which the logical calculus is susceptible, according as the terms represent concepts or propositions (p. 386). Why then, it may be asked, did Leibniz not succeed in definitively constructing the Algebra of Logic? M. Couturat finds the answer in Leibniz's preference for the intensive rather than the extensive view (pp. 386, 387), while the Algebra of Logic can only be based on the latter. That Leibniz failed to see this is, he thinks (p. 362), astonishing; for Leibniz clearly comprehended the difference between the two views, and, in a fragment of 1690, accurately defined and contrasted them, and even enunciated the law according to which extension varies inversely with intension (p. 362).

The final chapter (IX) deals with Leibniz's geometrical calculus, in which he attempts to free geometry from the consideration of magnitude, and, by an analysis of position (*Analysis situs*), express position directly (pp. 406, 427). What he succeeded in producing, however, was but a system of bipolar and tripolar coördinates (p. 428); and his lack of success was due to his failure to free himself from metrical considerations (pp. 412, 428, 438). That his idea of the geometrical calculus was, nevertheless, neither chimerical nor sterile, as so many philosophers and mathematicians have held, is shown, M. Couturat thinks, by the fact that Grassmann in his *Ausdehnungslehre* (1844) has successfully built upon Leibniz's foundations. As Boole rediscovered and realized one part, so Grassmann did another part of Leibniz's Universal Characteristic, and the two have revealed the fact that Leibniz's most daring conceptions were no idle dreams, but prophetic intuitions, anticipating by nearly two centuries the progress of science and of the human mind.

The brief Conclusion (pp. 431–441) brings together the more important intrinsic difficulties and defects in Leibniz's logical labors, which the exposition in the body of the book reveals, and declares that, however powerful and original Leibniz's mind may have been, he was in no sense the *autodidact* that he boasted of being; and that his great erudition and his reverence for the authority of Aristotle in logic and Euclid in mathematics, from whose influence he never fully freed himself, were the fundamental causes of his failure successfully to perfect himself in his logical calculus and his geometrical calculus. "It will never," remarks M. Couturat (p. 440), "be known how much such overperfect works as Aristotle's *Organon* and Euclid's *Elements* have cost the human mind, nor how many centuries they have retarded the progress of the sciences by discouraging innovators through their appearance of definitiveness."

This is not the place to discuss the merits of Leibniz's logical labors. As for M. Couturat's work, the scholarship and temper displayed throughout this book are worthy of the highest admiration; in these respects, it is a model of what such a book should be. Furthermore, the book is a contribution of the very greatest importance toward the correct understanding and the evaluation of Leibniz's philosophy. Certain it is that no one hereafter who undertakes to discuss the philosophy of Leibniz can ignore it. M. Couturat has conclusively proved that Leibniz's logical labors were far more extensive and intrinsically significant, and far more influential in shaping Leibniz's thought as a philosopher, mathematician, and theologian than has been hitherto realized; and he has given us a most full and able account of what these labors were in all their aspects. While greatly impressed with the strength of the evidence which the book also presents in support of the conclusion that Leibniz's logic was the real foundation and center of his whole system, the present reviewer is not prepared, without further investigation and an examination of the logical writings of Leibniz, published by M. Couturat since the appearance of this book, to regard that conclusion as fully established. In the case of a universal genius like Leibniz, it is easy to make out a strong case for this or that interest as being primary and central; for example, it would not be difficult to make out a plausible case (cf. p. 165, n. 2) in support of the view that Leibniz was primarily a theologian and that his logical and his mathematical labors alike were but auxiliary to his theology. This is said not to disparage the evidence adduced by M. Couturat in support of his revolutionary conclusion, but to lead to its being soberly weighed.

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Mind in Evolution. By L. T. HOBHOUSE. London, Macmillan & Co.; New York, The Macmillan Co., 1901.—pp. xv, 415.

The book has several claims to consideration: first, it is a fairly comprehensive review of the literature and status of comparative psychology. Mr. Hobhouse has made use of many authorities—even the most recent. Yet there are very singular omissions: for example, it is singular that James's name should not occur in a book which discusses instinct, habit, the limits and methods of acquisition, etc.

Second, it contains new experimental and critical matter. Mr. Hobhouse has endeavored to conduct experiments on animals (including monkeys) under conditions which more nearly fulfill the normal ones than many heretofore carried out (*e. g.*, Thorndike's). In this he is fairly successful, and his effective criticism of Thorndike, for example, is supported by experimental results. At the same time, the 'naturalness' of the conditions is certain to be criticised by the extreme advocates of exactness. The present reviewer's opinion, however, is on the side of Hobhouse and Mills — that artificiality in such experiments is the extreme to be avoided. Better 'natural history' observation in such a complex thing as an animal's learning processes, than an artificial exactness which paralyzes the learning process or ren-